Lecture 2

This document provides practice problems that are similar to those that will be asked during the final exam. Please note that the document reflects the style and not the number of the questions that will be on the exam.

Problem 1

Consider the following zero-sum game, where two players P_1 and P_2 can choose between actions A and B, and receive the payoff according to the following table:

		P_2: A	P_2: B
P_1	A	3	1
	B	4	5

For example, if P_1 selects action A and P_2 selects action B, then P_1 receives reward 1, while P_2 receives reward -1.

Which of the following statements are correct?

(a) The action profile where P_1 chooses A and P_2 chooses B corresponds to a Nash equilibrium.

(b) The action profile where P_1 chooses B and P_2 chooses B corresponds to a Nash equilibrium.

(c) The action profile where P_1 chooses B and P_2 chooses A corresponds to a Nash equilibrium.

(d) The game has only a Nash equilibrium if the two players are allowed to play mixed strategies.

Problem 2 (answers updated on December 20, 2021)

Consider the following zero-sum game where players P_1 and P_2 can choose between actions A and B and receive a payoff according to the following table:

		P_2: A	P_2: B
P_1	A	0.5	1
	B	3	x

where $x \in \mathbb{R}$. Which of the following statements are correct?

(a) There exists a Nash equilibrium with mixed strategies for any $x < 1$.

(b) For all $x \in \mathbb{R}$ there exists a unique Nash equilibrium.

(c) If P_2 plays according to the (Nash) equilibrium strategy, their strategy will be pure for $x \geq 1$.

Hint: Sketch the expected reward for both players, as we did in the lecture.

Problem 3 (question and answers updated on November 17, 2021)

Let $A \in \mathbb{R}^{2\times 2}$,

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{11}+c & a_{12}+c \end{pmatrix},$$

describe the rewards of a two-player zero sum game. For example, if Player 1 plays action 1 and Player 2 plays action 2, Player 1 receives reward a_{12}, whereas Player 2 receives reward $-a_{12}$. Both players play according to Nash equilibrium strategies.

Which of the following conditions are true for arbitrary $a_{11}, a_{12}, a_{11} \neq a_{12}$, and $c \neq 0$?

(a) Player 1 has a pure strategy.

(b) Player 1 has a strictly mixed strategy.

(c) Player 2 has a strictly mixed strategy.

(d) None of the above.

Problem 4

Let $A \in \mathbb{R}^{2\times 2}$ be given as

$$A = \begin{pmatrix} 0.5 & 1 \\ 2 & 0.5 \end{pmatrix},$$

and let

$$x^* := \arg\max_{x\in\Delta} \left(\min_{y\in\Delta} \left(x^\top A y \right) \right), \qquad y^* := \arg\min_{y\in\Delta} \left(\max_{x\in\Delta} \left(x^\top A y \right) \right),$$

where Δ denotes the two-dimensional unit simplex, that is, $\Delta := \{(x_1, x_2) \in \mathbb{R}^2 \mid x_1 \geq 0, x_2 \geq 0, x_1 + x_2 = 1\}$. Which of the following results is correct?

(a) $x^* = (3/4, 1/4), \quad y^* = (1/4, 3/4).$

(b) $x^* = (1/4, 3/4), \quad y^* = (3/4, 1/4).$

(c) $x^* = (2/3, 1/3), \quad y^* = (1/4, 3/4).$

(d) $x^* = (1, 0), \quad y^* = (0, 1).$

(e) None of the above.

Problem 5

Is the following statement correct: “Any two-player game with a finite number of actions admits a Nash equilibrium with mixed strategies”?

(a) Yes.

(b) No.

Problem 6

Consider a zero-sum game with two players and a finite number of actions which has a mixed Nash equilibrium. Is this equilibrium necessarily unique?

(a) Yes.

(b) No.